

Energy dependence of $\bar{K}N$ interactions and resonance pole of strange dibaryons

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We study the resonance energy of the strange dibaryons using two models with the energy-independent and energy-dependent potentials for the s-wave $\bar{K}N$ interaction, both of which are derived by certain reductions from the leading order term of the effective chiral Lagrangian. These potential models produce rather different off-shell behaviors of the two-body $\bar{K}N - \pi\Sigma$ amplitudes in $I = 0$ channel, i.e., the model with energy-independent (energy-dependent) potential predicts one (two) resonance pole in the $\Lambda(1405)$ region, while they describe the available data equally well. We find that the energy-independent potential model predicts one resonance pole of the strange dibaryons, whereas the energy-dependent potential model predicts two resonance poles: one is the shallow quasi-bound state of the $\bar{K}NN$, and another is the resonance of the πYN with large width. An investigation of the binding energy of the strange dibaryons will make a significant contribution to clarify resonance structure of s-wave $\bar{K}N - \pi\Sigma$ around the $\Lambda(1405)$ region.

Since the deeply bound kaonic nuclear states were predicted,¹⁾ few-nucleon systems with a kaon have attracted increasing interest. Among the deeply bound kaonic states, the resonances in the $\bar{K}NN - \pi YN$ ($Y = \Sigma, \Lambda$) system (strange dibaryons) are actively investigated recently, since the three-hadron dynamics can be theoretically handled by the well-established Faddeev equations. Using the coupled-channel Faddeev equations, two of the authors^{2),3)} and Shevchenko *et al.*⁴⁾ studied the possible existence and the energy of the strange dibaryons. It has been found in these studies that off-shell behaviors of the two-body $\bar{K}N - \pi\Sigma$ amplitudes in $I = 0$ channel, which involve the $\Lambda(1405)$ resonance, strongly affect the determination of the energy of the strange dibaryons.

In Refs. 2) and 3), the energy of the strange dibaryons is evaluated by employing the following energy-independent potential for the $\bar{K}N - \pi Y$ subsystem (we refer to this potential model as ‘E-indep’ throughout this letter):

$$V_{ij}^{\text{E-indep}}(p_i, p_j) = -C_{ij}(m_i + m_j). \quad (1)$$

Here indices i, j represent the meson-baryon channel in isospin basis; m_i (p_i) is the meson mass (the relative momentum in the center of mass frame) of the channel i ; $C_{ij} = \lambda_{ij} / (32\pi^2 F_\pi^2 \sqrt{\omega_i(p_i)\omega_j(p_j)})$ with $\omega_i(p_i) = \sqrt{m_i^2 + p_i^2}$ and $F_\pi = 92.4$ MeV. The coefficient λ_{ij} is determined by the flavor SU(3) structure constant, e.g., $\lambda_{\bar{K}N, \bar{K}N} = 6$, $\lambda_{\pi\Sigma, \pi\Sigma} = 8$ and $\lambda_{\bar{K}N, \pi\Sigma} = -\sqrt{6}$. The E-indep model predicts one resonance pole corresponding to the $\Lambda(1405)$, which is the quasi-bound state of the

$\bar{K}N$ state, and a virtual state of the $\pi\Sigma$ state (see below). The resulting strange dibaryon is found to be a deeply quasi-bound $\bar{K}NN$ state with the binding energy $B \sim 40 - 60$ MeV.

The energy-independent potential (1) is derived by a reduction²⁾ from the so-called Weinberg-Tomozawa term,⁵⁾ the leading order term of the chiral effective Lagrangian:

$$L_{\text{int}} = \frac{i}{8F_\pi^2} \text{tr}(\bar{\psi}_B \gamma^\mu [[\phi, \partial_\mu \phi], \psi_B]), \quad (2)$$

with ψ_B (ϕ) being the octet baryon field (the octet pseudoscalar meson field). It is also possible to construct a model of energy-dependent potential from Eq. (2) following the ‘on-shell factorization’ used in Ref. 6). The potential for the s-wave scattering is given by

$$V_{ij}^{\text{E-dep}}(p_i, p_j; E) = -C_{ij}(2E - M_i - M_j), \quad (3)$$

where E is the total scattering energy and M_i is the baryon mass of the channel i . The potential (3) depends explicitly on the meson-baryon scattering energy. (We refer to this potential model as ‘E-dep’.) The factor $m_i + m_j$ in Eq. (1) corresponds to the energy dependent factor $2E - M_i - M_j$ in Eq. (3). A common feature between the potentials of E-dep and the chiral unitary approach developed in Refs. 6), 7), 8), 9) is that the $\bar{K}N$ interaction depends explicitly on the scattering energy. The chiral unitary approach predicts two resonance poles of the $\Lambda(1405)$.⁸⁾

As shown below, the E-dep model indeed predicts two resonance poles in the $\Lambda(1405)$ region and has a very different off-shell behavior of the $\bar{K}N - \pi\Sigma$ amplitudes from the E-indep model, whereas both models reproduce the available data of the $\bar{K}N - \pi\Sigma$ reactions equally well. Our purpose of this letter is to examine how this different nature of the $\Lambda(1405)$ in the two potential models emerges in the energy of the strange dibaryon within the framework of the Faddeev equations.

Before examining the resonance poles in the $\bar{K}NN - \pi YN$ system, we first describe the two reaction models, E-indep and E-dep, for the $I = 0$ $\bar{K}N - \pi\Sigma$ subsystem. The amplitudes of the E-indep (E-dep) model is obtained by solving the Lippmann-Schwinger equations with $V_{ij}^{\text{E-indep}}(p_i, p_j)$ ($V_{ij}^{\text{E-dep}}(p_i, p_j, E)$) as a driving term. In the actual calculations, we regularize the loop integrals of the Lippmann-Schwinger equation with the dipole form factors $g_i(\vec{p}_i) = \Lambda_i^4/(p_i^2 + \Lambda_i^2)^2$, where the Λ_i is the cutoff parameter in the channel i .

It is noted that we will take the non-relativistic kinematics throughout this work. This is because the energy-dependent potential such as Eq. (3) causes a problem in the three-body calculations with the relativistic kinematics: Two-body scattering energy W_2 , which is defined by $W_2 = \sqrt{(W - E_{\text{sp}})^2 - p_{\text{sp}}^2}$ with the total scattering energy W and the spectator energy E_{sp} and momentum p_{sp} , becomes pure imaginary for large spectator momenta. With the non-relativistic kinematics such a difficulty can be avoided.

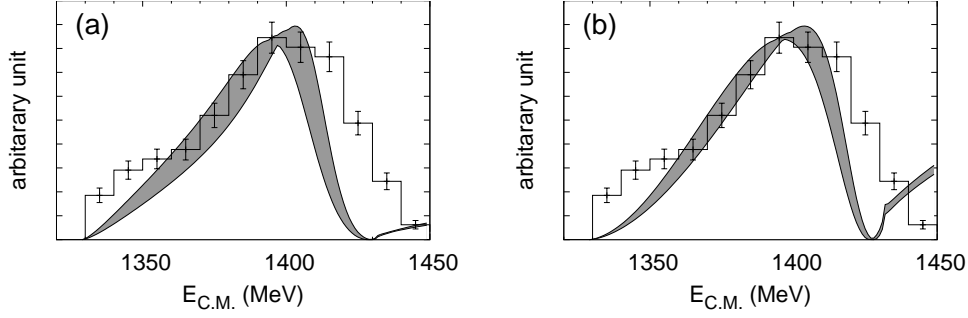


Fig. 1. The invariant mass distributions of the $I = 0$ $\pi\Sigma$ for (a) E-indep and (b) E-dep models.

We determine the cutoff parameters by fitting to the shape of the $\pi\Sigma$ invariant mass distribution in the $K^-p \rightarrow \pi\pi\pi\Sigma$ reaction¹⁰⁾ obtained with the assumption, $dN/dE \propto |t_{\pi\Sigma,\pi\Sigma}|^2 p_{\pi\Sigma}$ ($p_{\pi\Sigma}$ denotes the momentum in the $\pi\Sigma$ center of mass frame). The results of the fit are shown in Fig. 1 and the ranges of cutoff parameters are listed in Table I. It is noted that we only vary the parameters within a range of the uncertainty of the data.

With the parameters obtained by the fit, we also calculate the total cross sections of $K^-p \rightarrow \pi^0\Sigma^0$, to which only the $I = 0$ $\bar{K}N - \pi\Sigma$ system contributes. The resulting $K^-p \rightarrow \pi^0\Sigma^0$ total cross sections are shown in Fig. 2.

In Fig. 3 and Table I, we present the poles in the two-body amplitudes obtained from the E-indep and E-dep models. In the E-indep model (Fig. 3 (a)), we find one resonance pole, which corresponds to the $\Lambda(1405)$, in $(1405 - 1411) - i(14 - 20)$ MeV (R) on the $\bar{K}N$ physical and the $\pi\Sigma$ unphysical energy sheet. The pole R becomes

Table I. The cutoff parameters of the E-indep and E-dep models. In the forth column, the poles in the $\bar{K}N - \pi\Sigma$ amplitudes on the $\bar{K}N$ physical and the $\pi\Sigma$ unphysical energy sheet are also shown.

	$\Lambda_{\bar{K}N}$ (MeV)	$\Lambda_{\pi\Sigma}$ (MeV)	Poles (MeV)
E-indep	975 - 1000	675 - 725	$(1405 - 1411) - i(14 - 20)$, $(1296 - 1306) - i0$
E-dep	975 - 1000	675 - 725	$(1417 - 1423) - i(16 - 21)$, $(1335 - 1341) - i(65 - 79)$

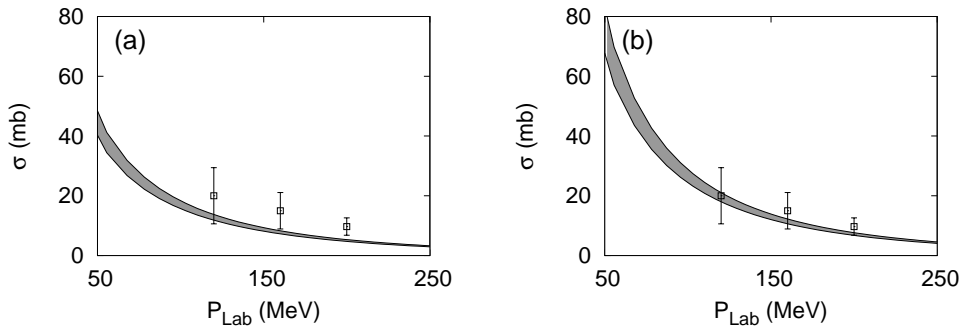


Fig. 2. The total cross section of the $K^-p \rightarrow \pi^0\Sigma^0$ reaction of (a) E-indep and (b) E-dep models

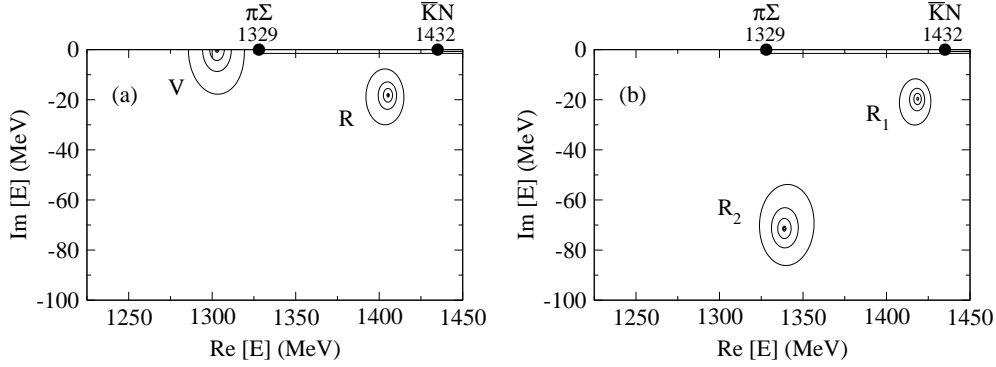


Fig. 3. Contour plots of the absolute values of t-matrix in $I = 0$ $\bar{K}N - \pi\Sigma$ system for (a) E-indep and (b) E-dep models with cutoff parameters $(\Lambda_{\bar{K}N}, \Lambda_{\pi\Sigma}) = (1000, 700)$ MeV. The $\pi\Sigma$ and $\bar{K}N$ threshold energy are shown together.

the $\bar{K}N$ bound state if we turn off the $\bar{K}N - \pi\Sigma$ transition potential. In addition, we find a virtual state in (1296 - 1306) MeV (V) on the same energy sheet.

On the other hand, the E-dep model has two resonance poles in the $\Lambda(1405)$ region in $(1417 - 1423) - i(16 - 21)$ MeV (R_1) and $(1335 - 1341) - i(65 - 79)$ MeV (R_2) as shown in Fig. 3 (b) and Table I. If the transition potentials between $\bar{K}N - \pi\Sigma$ channels are turned off, the poles R_1 become the $\bar{K}N$ bound states and R_2 become the resonances in the $\pi\Sigma$ channel. The similar result has been reported in Ref. 11).

We observe that the poles which strongly couple to the $\bar{K}N$ channel, R and R_1 , are rather stable against rather large variation of the parameters as far as they describe equally well the available data of the $\bar{K}N - \pi\Sigma$ scattering at low energies. On the other hand, the pole positions of V and R_2 which strongly couple to the $\pi\Sigma$ channel depend on reaction models. It is worthwhile to mention that the $\pi\Sigma$ scattering length may provide a useful constraint on the determination of the poles that strongly couple to the $\pi\Sigma$ channel.¹²⁾

Now let us discuss the resonance poles of the strange dibaryons in the E-indep and E-dep models. We shall show that the difference in the off-shell behavior of $\bar{K}N - \pi\Sigma$ amplitude strongly affects the resonance energy of the strange dibaryons in spin-parity $J^\pi = 0^-$ and isospin $I = 1/2$ channel.

In the three-body calculation, we start with the coupled-channel Alt-Grassberger-Sandhas (AGS) equations¹³⁾ for the $\bar{K}NN - \pi YN$ system. The equation is given by

$$X_{ij}(p_i, p_j; W) = C_{ij}^1 Z_{ij}(p_i, p_j; W) + \sum_{l,m} \int dq q^2 C_{il}^2 Z_{il}(p_i, q; W) \tau_{lm}(W) X_{mj}(q, p_j; W), \quad (4)$$

where i, j are specified by the Fock space of the three particles and the quantum

Table II. The cutoff parameters for the $I = 0$ and $I = 1$ $\bar{K}N - \pi Y$ scatterings, and the $I = 1/2$ and $I = 3/2$ πN scatterings. The values are given in MeV.

	$\Lambda_{\bar{K}N}^{I=0}$	$\Lambda_{\pi\Sigma}^{I=0}$	$\Lambda_{\bar{K}N}^{I=1}$	$\Lambda_{\pi\Sigma}^{I=1}$	$\Lambda_{\pi\Lambda}^{I=1}$	$\Lambda_{\pi N}^{I=1/2}$	$\Lambda_{\pi N}^{I=3/2}$
E-dep	975 - 1000	675 - 725	920	960	640	400	400
E-indep	975 - 1000	675 - 725	725	725	725	400	400

number of the interacting pair, and W is the three-body scattering energy. X , Z and τ denote the three-body amplitudes, particle-exchange potentials and isobar propagators, respectively. The coefficients $C^{1,2}$ are the spin-isospin re-coupling coefficients given in Ref. 2). We here define the two-body scattering energy W_2 in the three-body system as

$$W_2 = W - m_i - \frac{\vec{p}_i^2}{2\eta_i}, \quad (5)$$

where m_i , \vec{p}_i and η_i are the mass, momentum and reduced mass of the spectator particle i , respectively. As mentioned above, this non-relativistic energy helps to avoid the problem of the energy-dependent potentials. The cutoff parameters of the meson-baryon interaction for the E-indep and E-dep model are summarized in Table II. We take into account the Yamaguchi type of the baryon-baryon interactions for NN in 1S_0 channel²⁾ and YN in isospin $I = 1/2$ and $3/2$ channels.¹⁴⁾ It is noted that we again give ranges for the cutoff parameters of $\bar{K}N - \pi\Sigma$ channels. As a result, the energy of the strange dibaryons shown below is also presented with a range.

Solving the coupled-channel AGS equations (4) we find one resonance pole for the E-indep model on the $\bar{K}NN$ physical and πYN unphysical sheet. The resonance energy is $(B, \Gamma/2) = (44 - 58, 17 - 20)$ MeV, which is expressed with the binding energy B from the $\bar{K}NN$ threshold and the width Γ . In contrast, deeper binding energy ($B \sim 45 - 95$ MeV) of the strange dibaryon is predicted with the use of the relativistic kinematics in Ref. 2).

On the other hand, we find two resonance poles of the strange dibaryons for the E-dep model on the $\bar{K}NN$ physical and πYN unphysical sheet. One pole (Pole I) is found in $(B, \Gamma/2) = (9 - 16, 17 - 23)$ MeV, which is small binding energy and relatively small width. This implies that the strange dibaryon associated with Pole I will be observed as a shallow bound state. The similar result ($B \sim 20$ MeV) with the variational calculation was reported in Ref. 15). The other pole (Pole II) is found at $(B, \Gamma/2) = (67 - 89, 122 - 160)$ MeV. Since the imaginary part of Pole II is large, it may be hard to identify the resonance associated with Pole II in future experiments. Experimental studies of the binding energy of the strange dibaryon would give information on the subthreshold properties of the $\bar{K}N$ amplitude, which are not well constrained from the currently available data.

In summary, we have examined how the different nature of the $\Lambda(1405)$ emerges in the energy of the strange dibaryon within the framework of the Faddeev equations.

First we studied the resonance poles of the $\Lambda(1405)$ and the strange dibaryon using the models with energy-independent and energy-dependent potentials for the s-wave $\bar{K}N - \pi\Sigma$ interactions. The meson-baryon interactions have been derived from the Weinberg-Tomozawa term of the effective chiral Lagrangian. Only one resonance pole corresponding to the $\Lambda(1405)$ is predicted in the model with the energy-independent potential, whereas the energy-dependent potential model predicts the two resonance poles. The difference is the appearance of resonance pole which strongly couples to the $\pi\Sigma$ channel. The appearance of this resonance pole is attributed to the energy dependence of the potentials.

We then studied the strange dibaryons. The energy-dependent potential model predicts two poles of the strange dibaryons: one is the shallow bound state and another is the state with very large width. The latter state, which is originated in the two-body $\pi\Sigma$ resonance, however, may be difficult to be observed experimentally. Meanwhile the energy-independent potential^{2),3)} predicts the deeper binding energy than that of the energy-dependent potential model. Therefore, experimental determination of the binding energy of the strange dibaryon will make a significant contribution to clarify resonance structure of s-wave $\bar{K}N - \pi\Sigma$ around $\Lambda(1405)$ region.

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